Technical Notes

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Bow Wave Patterns

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Nomenclature

g = acceleration of gravity
L = ship length
L' = reference length
r = $\sqrt{x^2+y^2}$
s = phase function
s^0 = phase function of Kelvin wave
U = ship speed
ϕ = double potential
θ = $\tan^{-1}(y/x)$

Introduction

A METHOD of computing wave patterns from the dispersion relation for small Froude number $F=U/\sqrt{gL}$ has been developed by Keller.[3] Since the wavelength is proportional to U^2, the wavelength at low speed for small F is small compared to L. Hence, ray theory is appropriate for the present low-speed problem. An infinitely long cylinder with the cross section of the symmetric Joukowsky foil is employed as the ship and its trailing edge is taken to be the bow.

The dispersion relation is a first-order partial differential equation for the phase function s. The differential equation is solved by the method of characteristics. The so-called ray starts from the bow where waves are generated. The flowfield near the bow is examined to get the initial conditions of the characteristic equations. Then the equations are solved numerically. The wave pattern is obtained by tracing the points of the constant phase function s.

Formulation

We consider a stationary ship instead of the ship moving at constant speed U. We fix the origin of the rectangular coordinate system (x,y) on the surface of water at the bow such that the x axis is along the centerline of water plane. The relation between the local stream speed and the phase speed of local waves in the present steady problem is called the dispersion relation. The phase speed satisfies the dispersion relation,

$$(\nabla\phi\cdot\nabla s)^4=g^2(\nabla s)^2 \quad \text{on the surface of water} \tag{1}$$

where ∇ is the gradient in the x,y plane. Equation (1) indicates that the phase speed and the local stream speed in the opposite direction of the wave propagation are the same. Further s has to satisfy the boundary condition

$$\partial s/\partial n=0 \quad \text{around the ship boundary} \tag{2}$$

where $\partial s/\partial n$ is the normal derivative of s. Using some reference length L', we introduce the following dimensionless variables x', y', ϕ', and s' from the relations

$$\phi=UL'\phi', \quad (x,y)=L'(x',y'), \quad s=s'/F_0^2 \tag{3}$$

where $F_0=U/\sqrt{gL'}$. In terms of the dimensionless variables Eq. (1) becomes

$$(\nabla\phi\cdot\nabla s)^4=(\nabla s)^2 \tag{4}$$

by dropping the primes.

Double Potential

We consider an infinitely long cylinder with the cross section of the symmetric Joukowsky foil with a cusp at the trailing edge. By the Joukowsky transformation

$$\zeta=a-(z+\sqrt{z^2-4z})/2 \tag{5}$$

the foil is mapped onto a circle of the radius a in the ζ plane in Fig. 1. When $a=1$, the foil reduces to a flat plate extending from $x=0$ to $x=4$. The complex potential $W(z)$ of the flow around the ship is

$$W(z)=-(\zeta+a^2/\zeta) \qquad (a\geq 1) \tag{6}$$

Taking the real part of Eq. (6), we get the double potential ϕ in Eq. (4).

Characteristic Equations

The dispersion relation [Eq. (4)] in the polar coordinates (r,θ) is

$$(\phi_r s_r+\phi_\theta s_\theta/r^2)^4=s_r^2+s_\theta^2/r^2 \tag{7}$$

Equation (7) is solved for s by the method of characteristics. The characteristic equations of Eq. (7) are

$$\dot r=2s_r-4\phi_r(s_r\phi_r+s_\theta\phi_\theta/r^2)^3 \tag{8}$$

$$\dot\theta=2s_\theta/r^2-4\phi_\theta(s_r\phi_r+s_\theta\phi_\theta/r^2)^3/r^2 \tag{9}$$

$$\dot s_r=2s_\theta^2/r^3+4(s_r\phi_{rr}-2s_\theta\phi_\theta/r^3+s_\theta\phi_{r\theta}/r^2) \times(s_r\phi_r+s_\theta\phi_\theta/r^2)^3 \tag{10}$$

$$\dot s_\theta=4(s_r\phi_{r\theta}+s_\theta\phi_{\theta\theta}/r^2)(s_r\phi_r+s_\theta\phi_\theta/r^2)^3 \tag{11}$$

$$\dot s=-2(s_r^2+s_\theta^2/r^2) \tag{12}$$

Equations (8-12) constitute a system of five first-order ordinary differential equations in the five unknowns r, θ, s_r, s_θ, and s with the boundary condition $\partial s/\partial n=0$. For the initial conditions of s_r, s_θ, and s at the bow, the flow near the bow is examined. Near the bow we have

$$\phi(r,\theta)\sim r\cos\theta/a \qquad \text{for small } r \tag{13}$$

The flow near the bow is uniform. Therefore the initial wave at the bow is the Kelvin wave. Hence we find

$$s(r,\theta)\sim a^2 s^0(r,\theta) \qquad \text{for small } r \tag{14}$$

Received Nov. 20, 1981; revision received Aug. 23, 1982.

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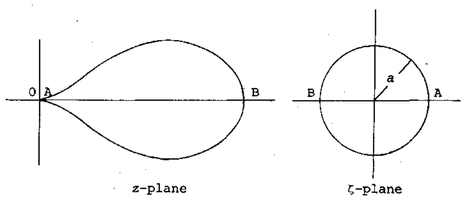

Fig. 1 Mapping.

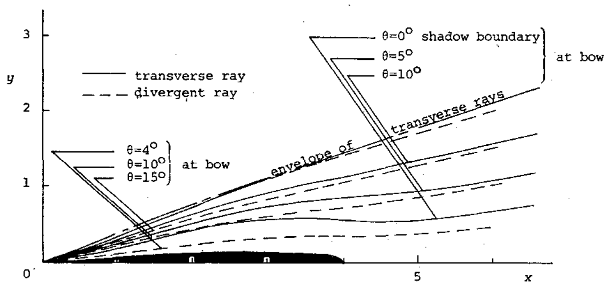

Fig. 2 Computed rays when a = 1.05.

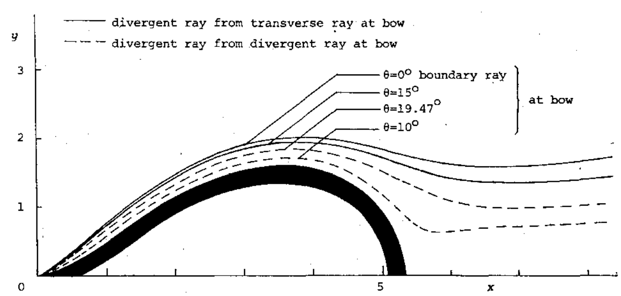

Fig. 3 Computed rays when $a=2$.

where $s^0(r,\theta)$ is the phase function of the Kelvin wave,

$$s^0(r,\theta) = -r\cos^{-2}\sigma(1+4\tan^2\sigma)^{-1/2}$$

$$\tan\sigma = (1 \pm \sqrt{1-8\tan^2\theta})/(4\tan\theta) \tag{15}$$

The Kelvin wave has two systems of waves, i.e., transverse waves and divergent waves. The positive sign in the second relation of Eq. (15) is for the divergent wave and the negative sign is for the transverse wave. The waves are confined inside a wedge the half angle of which is 19.47 deg. We now have two sets of rays starting from the bow, i.e., transverse rays for the transverse waves and divergent rays for the divergent waves. Equation (14) provides the initial values of s_r, s_θ, and s. The solution (r,θ) yields the path of ray and the wave patterns are the curves of constant s.

Results and Discussion

We first computed Eqs. (8-12) for the ray and wave pattern when $a=1$. The results show that all the rays are straight and the wave pattern is that of the Kelvin wave. When $a>1$, the ship form is determined from

$$z=2-(\zeta+1-a)-1/(\zeta+1-a) \quad \text{for } \zeta=ae^{i\sigma},\ 0\le\sigma<2\pi \tag{16}$$

In that case the computations indicate that the ray from Eqs. (8-12) moves away from the ship boundary or travels along it in the limiting case, i.e., the ray does not reflect on the ship boundary. Therefore the boundary condition $\partial s/\partial n=0$ is not necessary and can be ignored.

The computed rays are shown in Fig. 2 when $a=1.05$. Some transverse rays near 19.47 deg at the bow form an envelope.

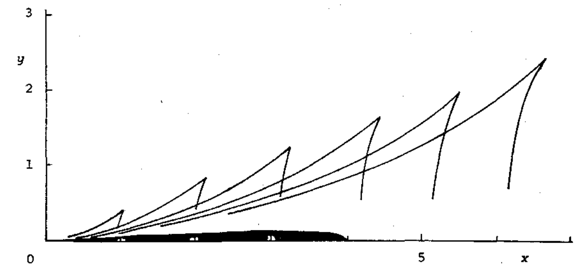

Fig. 4 Wave pattern when a = 1.05, s = −1, −2, −3, −4, −5, −6 from bow.

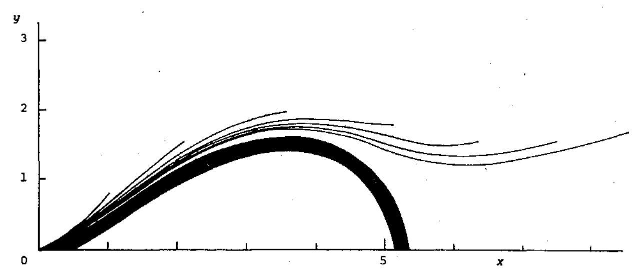

Fig. 5 Wave pattern when $a=2$, s = −1, −3, −5, −7, −9, −11, −13 from bow.

They remain transverse until they touch the envelope and become divergent rays when they leave the envelope. The transverse ray starting from $\theta=0$ deg at the bow forms a shadow region. Because waves diffract into the shadow region, the shadow region is not free of waves. The slope of the envelope at $r=20$ is close to that of 19.47 deg. Far downstream the flow is uniform and all the rays are straight.

We next consider the divergent rays. The sign of s_r of the divergent ray is initially negative while that of s_θ is positive. As θ tends to zero at the bow, the value of s_r tends to negative infinity and that of s_θ/r tends to positive infinity. Hence, it may not be possible to compute the divergent ray starting from $\theta=0$ deg at the bow. Along the divergent ray s_r changes its sign where the flow changes rapidly and recovers its original sign as the ray proceeds further downstream. It appears from the computations that the value of s_θ/r dominates that of s_r on the divergent ray and it becomes infinite on the limiting ray along the ship boundary. We now examine the divergent ray starting from $\theta=0$ deg at the bow in the limiting process to see if the ray forms a shadow region. From the dispersion relation [Eq. (4)] in the rectangular coordinates (x,y), we get the following ray equation:

$$\dot{x}=2s_x-4\phi_x(\phi_x s_x+\phi_y s_y)^3$$

$$\dot{y}=2s_y-4\phi_y(\phi_x s_x+\phi_y s_y)^3 \tag{17}$$

As a point (x,y) approaches the boundary from inside the flow, s_x and s_y tend to infinity. Hence the slope of the limiting ray is equal to that of the streamline along the ship boundary,

$$\mathrm{d}y/\mathrm{d}x=\phi_y/\phi_x$$

The divergent ray starting from $\theta=0$ deg at the bow follows the ship boundary. Thus the divergent ray does not form a shadow region as indicated in Figs. 2 and 3. The wave pattern is plotted in Fig. 4. As the value of a increases, the shadow region becomes larger. So long as the shadow region is present, the slope of the envelope far downstream is close to that of the boundary ray of the Kelvin wave. Beyond the certain value of a, the shadow region disappears completely and all of the rays are divergent except for those of the transverse waves at the bow. In that case, the envelope contracts almost to a point at the bow and all of the waves are divergent as indicated in Fig. 5.

Conclusion

There are three clear pictures of ship waves in Ref. 4. The ships in the pictures are blunt and seem to be running at high speed. In the second and third pictures, no transverse wave is seen. In the first picture, however, the transverse waves are seen downstream. It appears that the transverse waves come from the stern, not from the bow.

Because of the shadow region, even in the case of a slender ship the transverse waves are in the region between the envelope and the boundary ray, whereas the divergent waves are between the ship boundary and the envelope. Because the transverse waves diffract into the shadow region, the shadow region is not free of the transverse waves. Far downstream the flow becomes uniform. Hence all of the rays become straight and all waves are those of the Kelvin wave. The slope of the envelope far downstream may be the same as that of the half wedge angle, 19.47 deg of the Kelvin wave.

When the ship form is blunt, all of the waves from the bow are divergent. In that case the slope of the boundary ray far downstream is less than that of the 19.47 deg Kelvin wave. It should be noted that the present theory applies only at low speeds.

References

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Force and Moment in Incompressible Flows

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Introduction

THE central problem of external fluid dynamics is the prediction and/or measurement of the force and moment exerted upon a body immersed in an arbitrary flowfield by the surrounding fluid. Although complete knowledge (either analytical or numerical) of the flowfield allows evaluation of this force and moment by integrating, over the surface of the body, the elemental contributions due to pressure and viscous shear, it is certainly of great theoretical interest and, possibly, of practical usefulness to obtain general alternate expressions for such quantities. This is particularly true for the case of incompressible flows, of interest here, for which the elimination of pressure from the formulas expressing the force and moment is desirable, due to the peculiar character of this variable. At present, particular formulas are available, e.g., for the force acting on a sphere immersed in a viscous steady creeping flow (see Ref. 1, p. 233) and for the force acting on an arbitrary body immersed in a potential unsteady flow (see Ref. 2, p. 300). Moreover, for the case of inviscid flows past an arbitrary two-dimensional body, Blasius has derived general relationships for both the aerodynamic force and moment (see, e.g., Ref. 1, p. 433). However, more general expressions are not available to date and it seems worthwhile to attempt to derive formulas that are valid for any flow regime (i.e., for arbitrary values of the Reynolds number) and, therefore, are capable of reobtaining all previously mentioned results within a general framework and from a unified perspective, as done in the present Note.

Received April 23, 1982; revision received Aug. 2, 1982.

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Pressure Boundary Values Relationship

Let us consider an incompressible viscous flow which is governed by the time-dependent Navier-Stokes equations

$$\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}=-\nabla p-\nu\nabla\times\nabla\times\boldsymbol{u} \qquad (1)$$

$$\nabla\cdot\boldsymbol{u}=0 \qquad (2)$$

where $\boldsymbol{u}$ is the velocity vector and p the pressure (per unit density) of the fluid. Equations (1) and (2) are supplemented by the initial and boundary conditions

$$\boldsymbol{u}|_{t=0}=\boldsymbol{u}_0 \qquad (3)$$

$$\boldsymbol{u}|_S=\boldsymbol{b} \qquad (4)$$

where S is the boundary of the region V occupied by the fluid. The initial velocity field $\boldsymbol{u}_0$ is assumed to be solenoidal, i.e.,

$$\nabla\cdot\boldsymbol{u}_0=0 \qquad (5)$$

and the velocity $\boldsymbol{b}(\boldsymbol{r},t)$, prescribed at any point $\boldsymbol{r}\in S$, is assumed to satisfy the condition

$$\int \mathrm{d}S\boldsymbol{n}\cdot\boldsymbol{b}=0 \qquad (6)$$

which follows from Eq. (2) by the application of Green's formula, $\boldsymbol{n}$ denoting the outward normal unit vector. In what follows it will be assumed that the problem defined by Eqs. (1-6) has a solution $(\boldsymbol{u},p)$ and that the vector fields $\boldsymbol{u}$ and ∇p belong to L^2; L^2 denoting the Hilbert space of square summable vector functions, defined in V, i.e., fields $\boldsymbol{v}$ for which the integral $\int \mathrm{d}V|\boldsymbol{v}|^2$ is finite. Basic to the derivation of a relationship involving pressure boundary values is a decomposition theorem of the Hilbert space L^2 into three orthogonal subspaces (see Ref. 3, pp. 15-17) which allows elimination of the pressure at the boundary in favor of the velocity field. This is achieved by first projecting the momentum equation (1) onto one of the subspaces $(H_I\equiv\{\boldsymbol{v}\in L^2\,|\boldsymbol{v}=\nabla\eta,\ \nabla^2\eta=0\}$, i.e., by taking the Hibert scalar product of Eq. (1) by any $\nabla\eta\in H_I$, to obtain,

$$-\int \mathrm{d}V\nabla p\cdot\nabla\eta=\int \mathrm{d}V\left\{\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}+\nu\nabla\times\nabla\times\boldsymbol{u}\right\}\cdot\nabla\eta \qquad (7)$$

Then, by transforming the volume integral on the left-hand side of Eq. (7) into a surface integral by means of the equation $\nabla p\cdot\nabla\eta=\nabla\cdot(p\nabla\eta)$, and rewriting the linear terms in the right-hand side of Eq. (7) as surface integrals by virtue of the identities $\boldsymbol{u}\cdot\nabla\eta=\nabla\cdot(\boldsymbol{u}\eta)$ and $(\nabla\times\nabla\times\boldsymbol{u})\cdot\nabla\eta=\nabla\times\boldsymbol{\zeta}\cdot\nabla\eta=\nabla\cdot(\boldsymbol{\zeta}\times\nabla\eta)$, where $\boldsymbol{\zeta}=\nabla\times\boldsymbol{u}$ is the vorticity, the following